F.9 Metabolic Engines

Now want to explore how our heat engine analysis can be applied to animal metabolisms in general, and humans in particular.

At the most basic level, an engine:
1. derives energy from a heat source
2. converts some of it to work, with efficiency η
3. exports the rest of the heat to the environment

And animals/people go through a similar process, with one major difference
1. derives energy from a *food* source
2. converts some of it to work, with efficiency η
3. exports the rest of the heat to the environment

Both heat engines and metabolic engines are described by the first law: $-W_{done\,by\,engine} + Q = \Delta E$

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With respect to metabolic engines, these are the ramifications of each individual term:

$$-W_{you} + Q = \Delta E$$

W_{internal}
= work done by internal organs to stay alive
e.g., heart beating, lungs breathing, brain thinking

$$W_{\text{internal}} \sim (15\,\text{J/s})\Delta t$$

W_{external}
= work done against external environment,
e.g. walking, running, climbing, lifting weights,...

$$W_{walk} \sim 10\text{kcal/mile} \qquad \text{kcal} = 4180\text{J}$$

$$W_{run} \sim 20 \text{ kcal/mile}$$

$$W_{\text{lift}} \sim m_{weight}gh$$

$$W_{\text{climb}} \sim m_{you}gh$$

$$\frac{dQ}{dt} = \begin{cases} kA\dfrac{\Delta T}{\Delta x} & \text{conduction (through clothes)} \\ hA\Delta T & \text{convection (off skin)} \\ \sigma\varepsilon_{th}A\Delta T^4 & \text{thermal radiation (off skin)} \\ \varepsilon_{sol}IA_{\perp} & \text{solar radiation (into skin)} \\ \dfrac{dm}{dt}L_v & \text{evaporation (off skin)} \end{cases}$$

$$E_{\text{internal}} = KE_{\text{internal}} + PE_{\text{internal}}$$

$$= m_{you}c_{you}T + PE_{\text{chemical}}$$

$$c_{you} \sim 3.5\frac{\text{kJ}}{\text{kg}^\circ\text{K}}$$

Your efficiency is defined as the ratio of the work you do by the chemical potential energy you must metabolize to do it. For humans, it’s around 20% for most activities

$$\eta = \frac{W_{you}}{|PE_{chem}|} \sim 0.20$$

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In a nutshell,

- you ingest food, and metabolize the chemical potential energy.

- 20% of the energy goes into work (both $W_{internal}$ and $W_{external}$).

- the rest (80%) would show up as internal energy (which would increase your T and make you hot – not the attractive kind)

- Then hopefully you slough off most of this extra internal energy via some combination of the heat terms
  to keep your temperature close to 98F.

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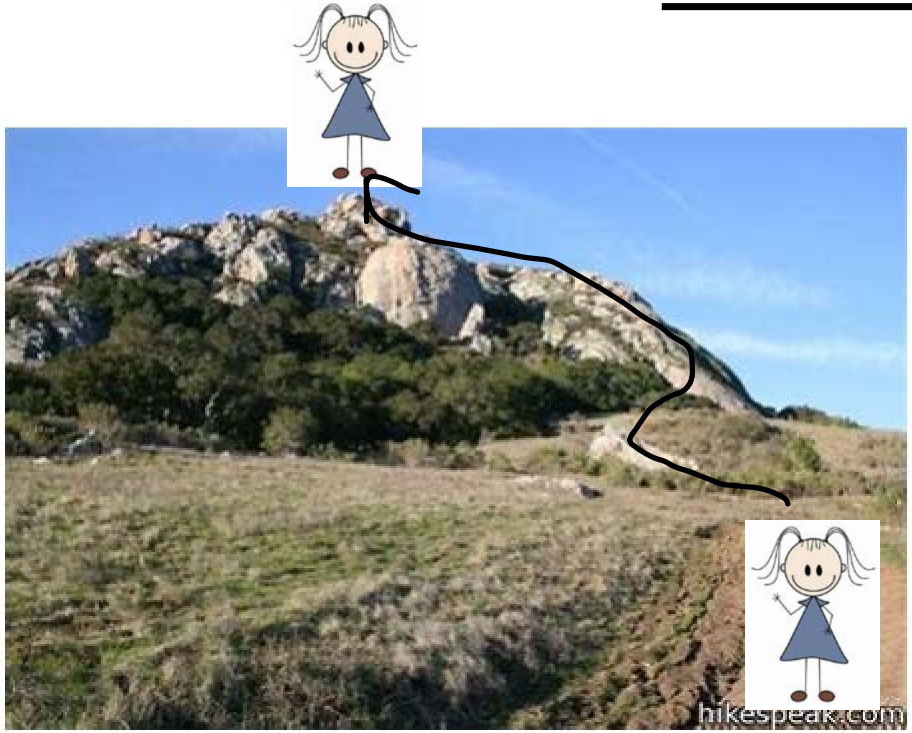

Suppose you (60kg) climb Bishop peak, which is about a 1.5mile hike, up around 400m, and maybe takes 40 minutes. What total work do you do?

$$W_{you} = W_{\text{internal}} + W_{\text{external}}$$

$$= W_{\text{internal}} + W_{\text{walk}} + W_{\text{lift yourself up 400m}}$$

$$= \left(15\frac{\text{J}}{\text{s}}\right)\Delta t + \left(10\frac{\text{kcal}}{\text{mile}} \times \#\text{miles}\right) + m_{you}gh$$

$$= \left(15\frac{\text{J}}{\text{s}}\right)(40 \times 60\text{s}) + \left(10\frac{4180\text{J}}{\text{mile}} \times 1.5\text{mile}\right) + (50\,\text{kg})(9.8\text{m/s}^2)(400\text{m})$$

$$= 36\text{kJ} + 63\text{kJ} + 196\text{kJ} = 295\text{kJ}$$

How much energy do you 'burn'?

This is a reference to the *chemical potential energy* that you must metabolize to do that work. We can get this using the relationship between work and efficiency:

$$\eta = \frac{W_{you}}{|PE_{chem}|}$$

$$0.20 = \frac{295\text{kJ}}{|PE_{chem}|}$$

$$|PE_{chem}| = \frac{295\text{kJ}}{0.20}$$

$$= 1480\text{kJ}$$

$$= 1480\text{kJ} \times \frac{\text{kcal}}{4.18\text{kJ}} = 354\text{kcal}$$

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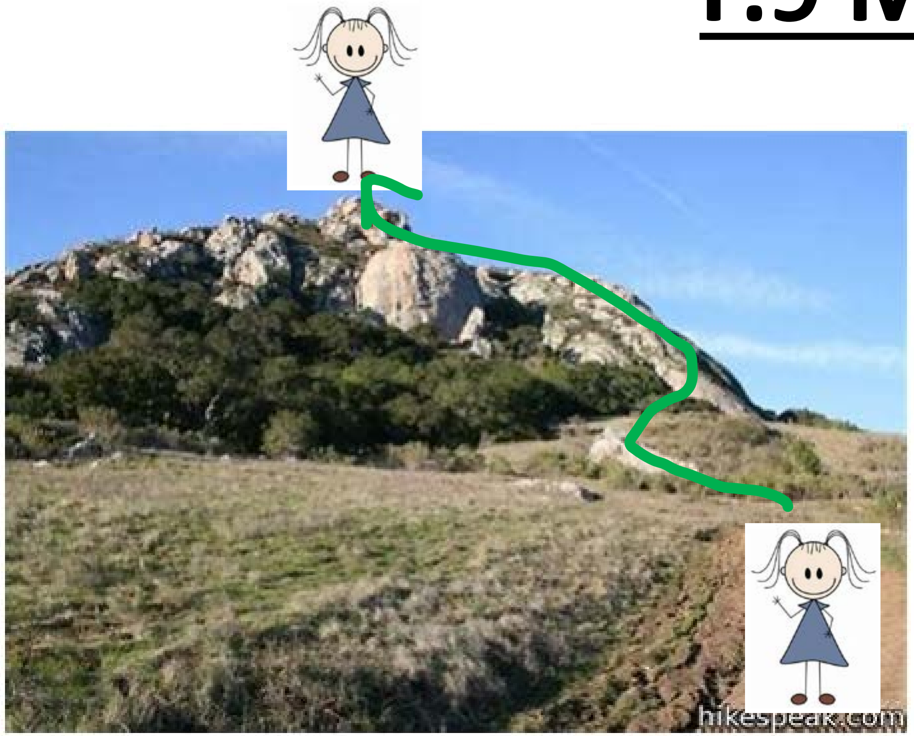

If you hiked up the mountain, wrapped in a plastic trash bag to eliminate all heat transfer, what would be your temperature increase?

We use the 1st law for this:

$$-W_{you} + Q = \Delta E_{\text{int}}$$

$$-295\text{kJ} + 0 = m_{you} c_{you} \Delta T + \Delta PE_{chem.}$$

$$-295\text{kJ} + 0 = (60\,\text{kg})\left(3.5\frac{\text{kJ}}{\text{kg}^\circ\,\text{C}}\right)\Delta T + (-1480\text{kJ})$$

note – sign: you always lose $\text{PE}_{\text{chem.}}$

$$\Delta T = \frac{1480\text{kJ} - 295\text{kJ}}{(60\,\text{kg})\left(3.5\frac{\text{kJ}}{\text{kg}^\circ\,\text{C}}\right)} = 5.6^\circ\text{C} = 10.1^\circ\text{F}$$

Again, your temperature increases because only 20% of the energy burned goes into work. The other 80% has gone into your internal kinetic energy: mcΔT. And with the plastic bag on, you can't transfer this energy to the outside air.

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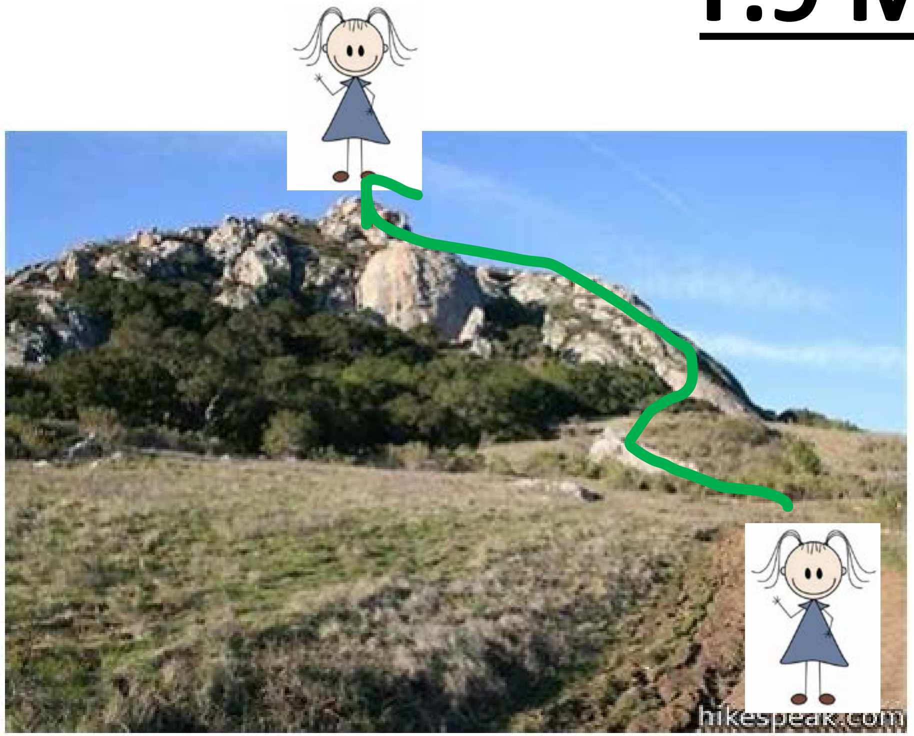

Now let's assume that you like to hike wild and free, and so *do* transfer heat, by thermal radiation and evaporation. How much water do you evaporate?

Let: $\varepsilon_{th} = 1$,
your surface area $A = 1.6\text{m}^2$,
outside air temperature be 80F = 300K,
your temperature to be 98F = 310K.

Again, we use the 1st law:

$$-W_{you} + Q = \Delta E_{\text{int}}$$

$$-W_{you} + Q_{thermal} + Q_{evaporation} = m_{you} c_{you} \Delta T + \Delta PE_{chem.}$$

$$-W_{you} + \varepsilon_{th} \sigma A (T_{air}^4 - T_{you}^4) \Delta t + L_v \frac{dm_{water}}{dt} \Delta t = m_{you} c_{you} \Delta T + \Delta PE_{chem}$$

$$-295\text{kJ} + (1)\left(5.67\times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot {}^{\text{o}}\text{K}^4}\right)(1.6\text{m}^2)\left[(300\text{K})^4 - (310\text{K})^4\right](40\times 60\text{s}) + \left(2256 \frac{\text{kJ}}{\text{kg}}\right)\Delta m_{water} = m_{you} c_{you}(0) + -1480\text{kJ}$$

$$-295\text{kJ} + -247\text{kJ} + \left(2256 \frac{\text{kJ}}{\text{kg}}\right)\Delta m_{water} = -1480\text{kJ}$$

$$\Delta m_{water} = -0.41\text{kg}$$